Clique percolation method: memory efficient almost exact communities

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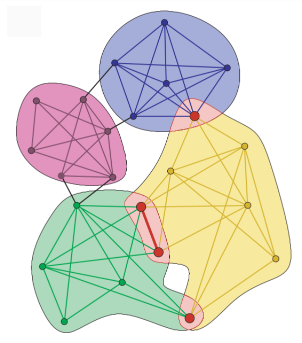
Contents

Parts of the presentation :

Communities in a graph

Community :

- Densely connected inside
- Sparsly connected outside



Palla et al. 2005

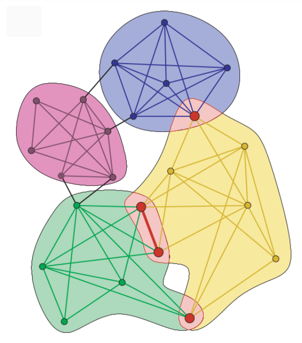
Communities in a graph

Community :

- Densely connected inside
- Sparsly connected outside

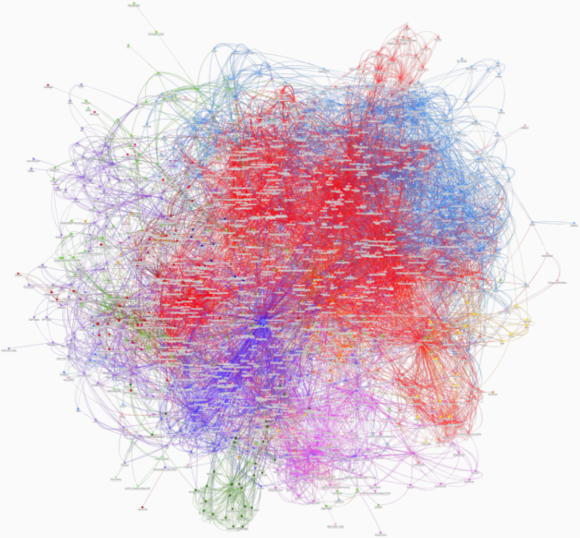
Interest :

- Massive graph : zoom in and out
- Biological interactions
- Content recommendation
- ...



Palla et al. 2005

Scaling up to massive graph



CPM - Definition

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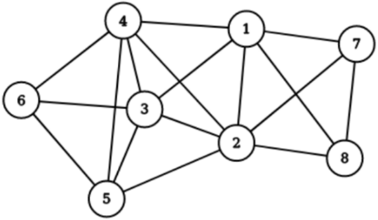

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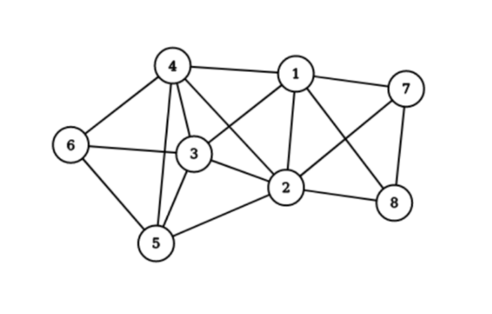

k-clique

Set of *k* nodes all connected to each other.

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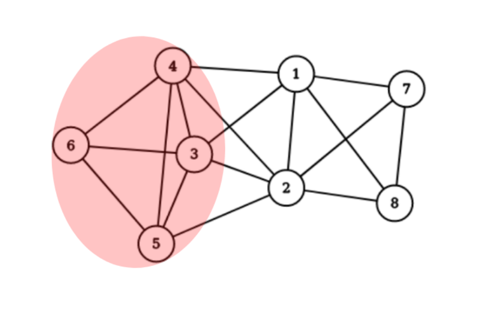

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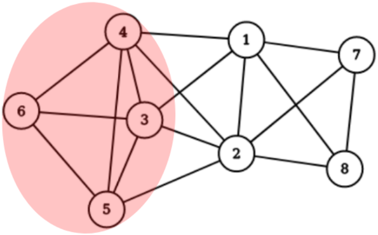

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Adjacent k-cliques

Two k-cliques are adjacent if they share $k-1$ nodes (a $(k-1)$-clique).

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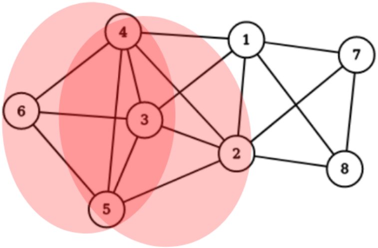

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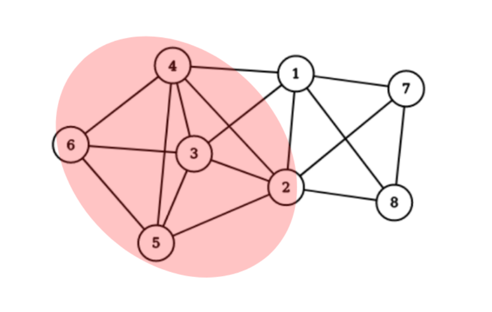

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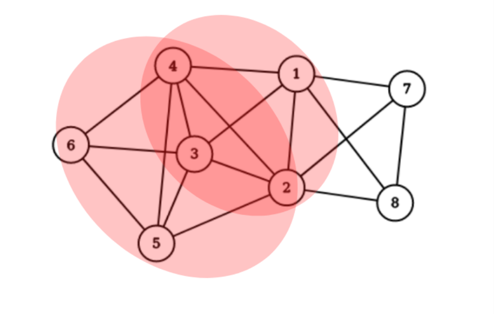

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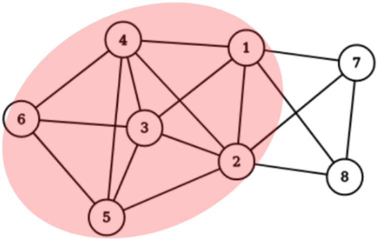

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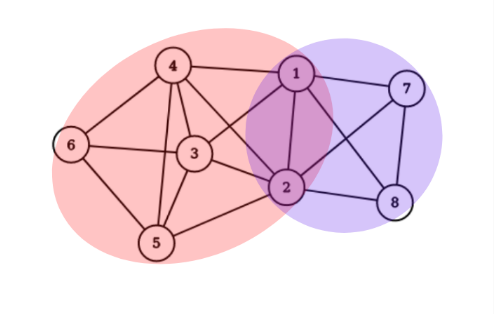

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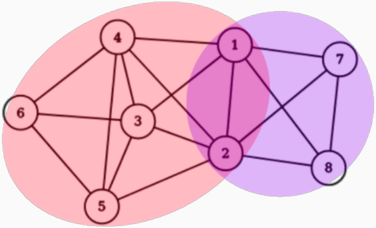
CPM Community

Maximal set of adjacent k-cliques.



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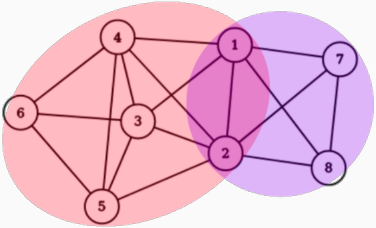


Literature

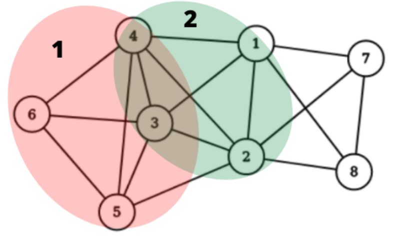
- Palla *et al.* **2005** : first definition
- Kumpula *et al.* **2008** : based on k-clique enumeration
- Reid *et al.* **2012** : based on maximal clique enumeration
- Gregori *et al.* **2013** : parallel version, based on maximal cliques

Exact and approximate algorithms

Exact Algorithm

Exemple :

$k = 4$



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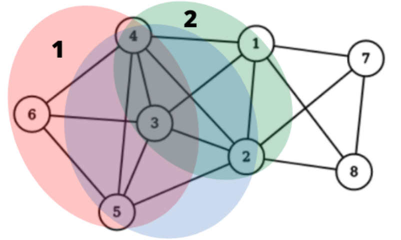

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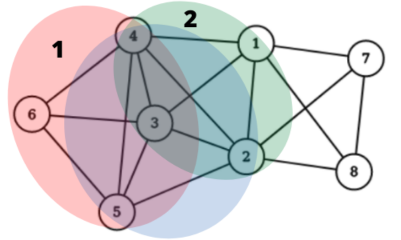

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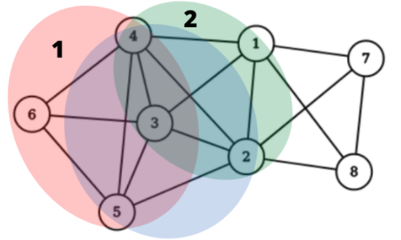

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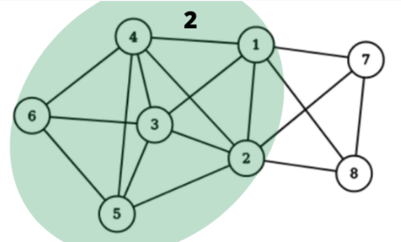

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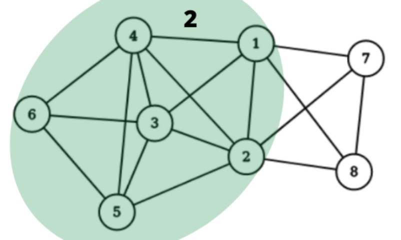

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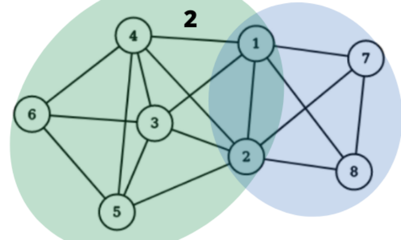

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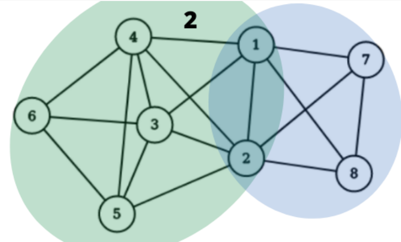

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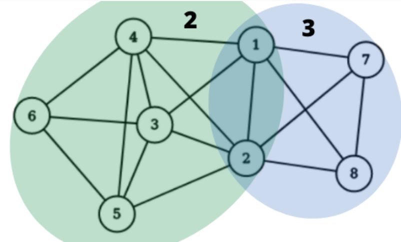

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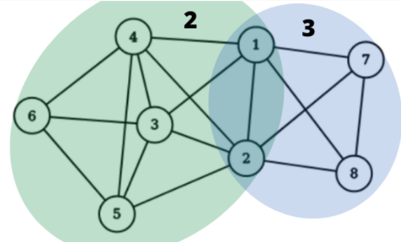

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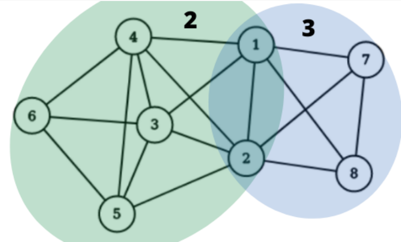

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- Update with **Union-Find** datastructure

Exact Algorithm

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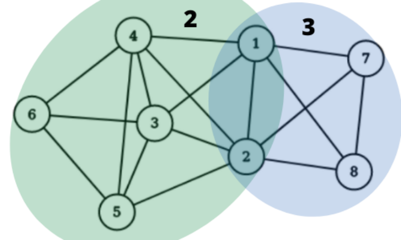

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- Update with **Union-Find** datastructure
- Complexity per k-clique : $\mathcal{C}_k \approx \mathcal{O}(k)$

Approximate Algorithm

Limit on massive graphs

Massive graph : the larger k is, the more k-cliques there are.

We work on $k \sim 3 - 15$.

Reduce memory

- Exact algorithm : Store all $\underline{(k-1)\text{-cliques}}$.
- Approximate algorithm : Store all $\underline{z\text{-cliques}}$, $z < k - 1$.

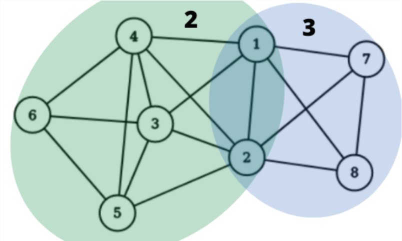
Principle of the Approximate Algorithm

$(k-1)$-clique $\approx$ the set of its z-cliques.

Approximate Algorithm

Example :

$k = 4, z = 2$



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- Update with **Union-Find** datastructure
- Complexity per k-clique : $\mathcal{C}_k \approx \mathcal{O}\left(k \cdot \binom{k-1}{z}\right)$

Exact and approximate algorithms

Result of Approximate Algorithm

Some exact communities are merged together.

Accuracy between Exact and Approximate Algorithm

Tool : ONMI *(McAid et al 2013)*

On all the experiments :

$\underline{z=2}$	$\underline{z=3}$
$> 93.8\%$	$> 99.5\%$
Mean : 98.6%	Mean : 99.95%
Median : 99.4%	Median : 100%

Conclusion

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- Exact algorithm, gain of time ;
- Almost exact algorithm, gain of memory
  $\Rightarrow$ best scale ;
  $\Rightarrow$ accurate approximate communities.

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Perspectives

- can bad merges of communities be avoided ?
- order of k-cliques on the accuracy of the result ;
- link streams ;